Results

1. **Magnetic curves in the generalized Heisenberg group**, *Marian Ioan Munteanu, Ana Irina Nistor,* Nonlinear Analysis (theory methods and applications) 214 (2022), art. nr. 112571. ISSN: 0362-546X

   We study the magnetic trajectories in the generalized Heisenberg group H(n,1) of dimension (2n+1) endowed with its quasi-Sasakian structure. We prove that the trajectories are Frenet curves of maximum order 5 and we completely classify them.

   If on a Riemannian manifold we consider a magnetic field, represented by a closed 2-form, the corresponding trajectories are called magnetic curves or magnetic geodesics. Both, geodesics and magnetic curves, come from variational problems, so the importance of the latter leads to a deep study (both by analogy with geodesics and on its own). I was interested in the study of magnetic curves in the generalized Heisenberg space H(n,1), which is canonically endowed with a quasi-Sasakian structure. These curves have an interesting property, namely that they form a constant angle with the Reeb vector of H(n,1). We then showed that a magnetic curve in H(n,1), which is not geodesic, has maximum order 5. This result together with other related results, makes us believe that the result is valid in any quasi-Sasakian manifold of size strictly greater than 3. In addition, all 4 non-zero curves are constant, so the magnetic curve is a slant helix. The obtained results were included in a paper entitled "Magnetic curves in the generalized Heisenberg group".

2. **Warped product hypersurfaces in pseudo-Euclidean space**, *Moruz Marilena,* *https://arxiv.org/abs/2208.07726*

   We study hypersurfaces in the pseudo-Euclidean space E^(n+1)_s , which write as a warped product of a 1-dimensional base with an (n−1)-manifold of constant sectional curvature. We show that either they have constant sectional curvature or they are contained in a rotational hypersurface. Therefore, we first define rotational hypersurfaces in the pseudo-Euclidean space. We give the following result:

   Theorem Let M^n = I ×_f \tilda{M}^{n−1}(c) be a warped product of a real interval I and an (n-1)-dimensional real space form \tilda{M}^{n−1}(c) with constant sectional curvature c, for a positive, non-constant function f on \mathbb{R}. Suppose that F:M^n \to E^(n+1)_s is a non-degenerate isometric immersion into the (n + 1)-dimensional pseudo-Euclidean space, endowed with an indefinite metric of positive signature s. Then M^n has constant sectional curvature or is a rotational hypersurface.

3. **On the nonexistence and rigidity for hypersurfaces of the homogeneous Nearly Kähler S^3 x S^3. Zejun** Hu, Marilena Moruz, Luc Vrancken, Zeke Yao, Differential Geometry and its Applications, 75, 2021, 101717.

   In this paper, we study hypersurfaces of the homogeneous NK (nearly Kähler) manifold S3×S3. As the main results, we first show that the homogeneous NK S3×S3 admits neither locally conformally flat hypersurfaces nor Einstein Hopf hypersurfaces. Then, we establish a Simons type integral inequality for compact minimal hypersurfaces of the homogeneous NK S3×S3 and, as its direct consequence, we obtain new characterizations for hypersurfaces of the homogeneous NK S3×S3 whose shape operator A and induced almost contact structure φ satisfy Aφ =φA. Hypersurfaces of the NK S3×S3 satisfying this latter condition have been

classified in our previous joint work (Hu et al. 2018).

4. **Totally geodesic surfaces in the complex quadric.** Marilena Moruz, Joeri Van der Veken, Luc Vrancken, Anne Wijffels, Contemporary Mathematics, 2022, 777, ISBNs: 978-1-4704-6015-0 (print); 978-1-4704-6874-3 (online). DOI: https://doi.org/10.1090/conm/777.

   We provide explicit descriptions of all totally geodesic surfaces of a complex quadric of arbitrary dimension. Totally geodesic submanifolds of complex quadrics were first studied by Chen and Nagano in 1977 and fully classied by Klein in 2008. In particular, we interpret some of these surfaces as Gaussian images of surfaces in a unit three-sphere and all others as elements of the Veronese sequence introduced by Bolton, Jensen, Rigoli and Woodward.
   We also briefly discuss how the classication can be translated to the noncompact dual of the complex quadric, namely the hyperbolic complex quadric.

5. **Magnetic Jacobi fields in Sasakian space forms.** Jun-Ichi Inoguchi, Marian Ioan Munteanu. In peer review process.

   The present paper is a continuation of the paper *Magnetic Jacobi Fields in 3-Dimensional Sasakian Space Forms. Jun-Ichi Inoguchi, Marian Ioan Munteanu, J. Geom Anal 32, 96 (2022). https://doi.org/10.1007/s12220-021-00851-6*, for some arbitary dimension (odd). It is very difficult to study the Jacobi magnetic fields of non-uniform magnetic fields in an arbitrary Riemannian manifold endowed with a magnetic field. The canonical magnetic fields of Sasakian manifolds are non-uniform but exact. In this paper we show that the Jacobi magnetic fields can be completely determined on Sasakian space forms of dimension greater than or equal to 5. The paper is in the peer-review process at a specialized journal.

6. **Ruled Real Hypersurfaces in the Indefinite Complex Projective Space**. Marilena Moruz Miguel Ortega, Juan de Dios Pérez, Results Math 77, 147 (2022). https://doi.org/10.1007/s00025-022-01691-8 .
   The main two families of real hypersurfaces in complex space forms are Hopf and ruled. However, very little is known about real hypersurfaces in the indefinite complex projective space C_p^n . In a previous work, Kimura and the second author introduced Hopf real hypersurfaces in *C_p^n* . In this paper, ruled real hypersurfaces in the indefinite complex projective space are introduced, as those whose maximal holomorphic distribution is integrable, and such that the leaves are totally geodesic holomorphic hyperplanes. A detailed description of the shape operator is computed, obtaining two main different families. A method of construction is exhibited, by gluing in a suitable way totally geodesic holomorphic hyperplanes along a non-null curve. Next, the classification of all minimal ruled real hypersurfaces is obtained, in terms of three main families of curves, namely geodesics, totally real circles and a third case which is not a Frenet curve, but can be explicitly computed. Four examples of minimal ruled real hypersurfaces are described.

7. **Totally Geodesic surfaces in the Nearly Keahler S^3 x S^3.** Moruz Marilena

   The study of total geodesic surfaces in the Nearly Kaehler S^3 x S^3 manifold followed from the approach of the second problem in the initial plan of the research project.
   The main result of this work consists in the classification theorem of totally geodesic surfaces in the Nearly Kaehler manifold S^3 x S^3. The proof of this result is technical, several cases are distinguished that we consider depending on the size of the vector space

generated by the tangent vectors of the surface together with the main structures defined in S^3 x S^3. An essential role in the research technique used is the knowledge of the Riemannian curvature tensor of the ambient space. A special case is distinguished when the surfaces are, in addition, almost complex: for a vector field X tangent to the surface we have immediatelly determined a tangent vector field given by JX, where J is the almost complex structure on S^3 x S^3. This case is already studied in the literature (J. Bolton, F. Dillen, B. Dioos, L. Vrancken, Z. Hu, Y. Zhang). The existence of total geodesic surfaces for the other (dimensional) cases is completely determined in this paper. The completion of the article depends on the illustration of some examples for the cases of the classification theorem, which is being worked on, and on the writing of the obtained results.

8. **Magnetic Jacobi Fields in 3-Dimensional Sasakian Space Forms** Jun-Ichi Inoguchi, Marian Ioan Munteanu, J. Geom Anal 32, 96 (2022). https://doi.org/10.1007/s12220-021-00851-6
   Representative examples of uniform magnetic fields are furnished by Kähler magnetic fields. From this point of view, magnetic Jacobi fields on surfaces or Kähler manifolds were investigated by Adachi and Gouda.On the contrary, Sasakian manifolds have nonuniform magnetic fields.We obtain all magnetic Jacobi fields along contact magnetic curves in 3-dimensional Sasakian space forms.

9. **Magnetic Geodesic in (Almost) Cosymplectic Lie Groups of Dimension 3.** Marian Ioan Munteanu, Mathematics 2022, 10, 544. https://doi.org/10.3390/math10040544

   In this paper, we study contact magnetic geodesics in a 3-dimensional Lie group G endowed with a left invariant almost cosymplectic structure. We distinguish the two cases: G is unimodular, and G is nonunimodular. We pay a careful attention to the special case where the structure is cosymplectic, and we write down explicit expressions of magnetic geodesics and corresponding magnetic Jacobi fields.

10. **Magnetic curves in quasi-sasakian manifolds of product type.** Marian Ioan Munteanu, Ana Irina Nistor, New Horizons in Differential Geometry and its Related Fields, pp. 1-22 (2022). https://doi.org/10.1142/9789811248108_0001

    In this paper we give an armative answer to sustain the conjecture about the order of a magnetic curve in a quasi-Sasakian manifold. More precisely, we show that the magnetic curves in a quasi-Sasakian manifold obtained as the product of a Sasakian and a Kaehler manifold have maximum order 5. Moreover, we obtain the explicit parametrizations, the periodicity conditions and examples in the study of magnetic curves in S^3\times S^2.