LAUDATIO

In honor of **John Horton CONWAY**,

John von Neumann Professor, Emeritus, Princeton University

On the occasion of the award of the Doctor Honoris Causa title of
Alexandru Ioan Cuza University of Iaşi

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Education. John Conway is born in Liverpool in 1937. Mathematics is present in his life from his very early years: at the age of 4 he can enumerate the powers of 2; seven years later, in an interview for secondary school, his answer to the question, “What do you want to be when you grow up?” is “A mathematician at Cambridge” – which he achieved at age 27. During all his education in school and later on at Gonville and Caius College, Cambridge, his performance in Mathematics is outstanding, as it is in other subjects such as Astronomy.

John Conway obtains his BA in 1959 and is awarded his doctorate in 1962 at *Gonville and Caius College* for the thesis “Homogeneous ordered sets,” supervised by Harold Davenport. The same year, he is appointed College Fellow and Lecturer in Pure Mathematics at the University of Cambridge. In 1975, John Conway is promoted to reader in Pure Mathematics and Mathematical Statistics, and in 1983, he becomes a Professor at Cambridge. In 1987, he is appointed to the John von Neumann Chair of Mathematics at Princeton University.

As a student at Cambridge, John Conway undertakes research in *number theory*, proving, in his PhD thesis, the conjecture by the 18^{th}-century English mathematician Edward Waring that every integer can be written as the sum of 37 numbers, each raised to the fifth power. He is also interested in infinite ordinals. During his studies, he developed his interest in games. As John Conway says, “I used to feel guilty in Cambridge that I spent all day playing games while I was supposed to be doing mathematics. Then, when I discovered surreal numbers, I realized that playing games IS mathematics.”

Conway's scientific contributions. It is difficult, indeed daring, to summarize Conway's comprehensive contribution to Mathematics and its applications. We confine ourselves to presenting chronologically a few of his groundbreaking achievements, merely summarizing some of the numerous remaining ones.

Group Theory. The fascinating story of the classification of finite simple groups began when Evariste Galois (1832) introduced the notion of a normal subgroup and found the first examples of simple groups. Finite groups are the mathematical abstraction of symmetry. For nearly 200 years, some of the most brilliant mathematicians have contributed to the construction of what eventually became the classification theorem of finite simple groups. This states that every finite simple group either belongs to one of three types (cyclic group with prime order; alternating group of degree at least 5; simple group of Lie type) or is one of the 26 *sporadic* simple groups (in clustering, the latter would be called *outliers*). The proof of this theorem, developed mostly between 1955 and 2004, spans tens of thousands of pages in hundreds of journal articles written by some 100 authors.

A decisive contribution towards this classification theorem was to mark the spectacular start of John Conway's acclaimed career as one of the outstanding mathematicians of our time. John Conway discovered three of the 26 sporadic groups – known as Co1, Co2 and Co3 – and all but two of the sporadic groups known by then can be obtained as the homomorphic image of subgroups of Co1. In Martin Gardner's words, "This is a breakthrough that has had exciting repercussions in both group theory and number theory."

In 1965, the Leech lattice was shown to provide a dense packing of hyperspheres in 24 dimensions. John Leech asked Conway, among others, to investigate the corresponding symmetry group. Mainly during one 12-hour session, Conway showed that the group of automorphisms of the Leech lattice, now known as Co_0 (its order is 8,315,553,613,086,720,000), when factored by its center, gives a finite simple group that was unknown at the time. This is the largest of the Conway groups, $\mathbf{Co_1}$, of order 4,157,776,806,543,360,000. Conway found two other new finite simple groups, $\mathbf{Co_2}$ (of order 42,305,421,312,000) and $\mathbf{Co_3}$ (of order 495,766,656,000), which are isomorphic to subgroups of Co_1.

In 1969 the first volume of the *Bulletin of the London Mathematical Society* published the full details of Conway's landmark discovery. Finite simple groups are instrumental in studying groups in general and their applications are diverse and often spectacular. We mention here merely the relevance of sphere packings and simple groups to error-correcting codes. Over 15 years, Conway, together with former doctoral students from Cambridge – Robert Curtis, Simon Norton, Richard Paker and Robert Nilson – produced a complete list of all finite simple groups; the result was his 1985 book *The Atlas of Finite Groups: Maximal Subgroups and Ordinary Characters for Simple Groups.*

John Conway is a not only a pioneer but also a visionary. In 1979, Conway, together with Simon Norton, made the famous "Monstrous moonshine conjecture" that relates discrete and non-discrete Mathematics. The conjecture was eventually proved in 1992 by Richard

Borcherds, who had been a Conway PhD student. The conjecture describes “the very strange connections” between the monster group *M* and modular functions. The monster group M is the largest sporadic finite simple group – more than 8*10**53 symmetries – containing as subquotients 20 of the 26 sporadic groups. Modular functions, and in general modular forms, appear in mathematical fields such as complex analysis (where they belong), algebraic topology and string theory. Thus, the now-proved conjecture ties the monster group M to various aspects of Mathematics and Mathematical Physics. Conway says, “The Monster is supported by a geometrical object in 196,883 dimensions; I would love to understand why the Monster is there”.

The Game of Life. John Conway became widely known beyond the world of pure mathematics with his invention of a simple universal cellular automaton, which became hugely popular as the 0-players *Game of Life*. In conceiving it, Conway aimed at simplifying John von Neumann’s search of a *universal cellular automaton*. Von Neumann had devised a mathematical model involving intricate rules on a Cartesian grid with neighborhoods of five cells and 29 states. Conway managed to simplify von Neumann’s ideas into his *Game of Life*, which has basically three rules and two states. After a long elaboration on Go boards, Conway described the game to his friend Martin Gardner who, in turn, explained it in his October 1970 Mathematical Games column in *Scientific American.* As Martin Gardner later on put it, “The game made Conway instantly famous, but it also opened up a whole new field of mathematical research, the field of cellular automata.”

Thousands of *Life* programs are currently available online and new discoveries are reported (for example, in 2013, the first replicator in Conway's Game of Life was reported on Conwaylife.com; it produces a full copy of itself, including the instruction tape). But, while popularity may be interesting, *Life* has profound and far-reaching properties. To start with, it is *computationally universal*: it can simulate any single-taped Turing machine. In other words, *The Game of Life* is theoretically as powerful as any computer with no memory/time constraints. Any question that can be answered to using Mathematics can be phrased as "Will a particular configuration of *Life* last indefinitely or not?" (*Winning Ways for Your Mathematical Plays* by Conway, Guy and Berlekamp, 1982). In *Life* a "universal constructor" containing a Turing complete computer can be built that can produce copies of itself. On the other hand, by visualizing emergence (a complex behaviour arising from few and very simple rules), self-organization and self-replication in an accessible manner, the *Game of Life* is a paradigmatic tool for scientists in Economics, Computer Science, Biology or generative science in general. *The Game of Life* and other cellular automata have been used, for example, to model the role of DNA in transmitting information from one generation to the next. Through *Life*, the field of Artificial Life made a decisive leap forward.

Surreal numbers. A truly astonishing discovery was Conway's surreal numbers – a (now) straightforward completion of the system number containing integers, rationals, reals, complex and transfinite numbers. Real numbers were accepted for 200 years as the basis for developing any further number system. Conway came up with a simple and deep question: *What if every two-person game is a number?* (in a way, similar to the idea behind Gödel numbers). The discovery came out of attempts to analyze the game of Go rather than of undertaking the abstract task of developing number systems. Conway noticed that, near its completion, a game could be seen as the sum of smaller games; this supported the idea that, from a certain point of view, some games have similarities with numbers; it is from this observation that the surreal numbers system emerged (or, rather, was unveiled).

The surreal number system is an arithmetic continuum with the arithmetic properties of a (n ordered) field. It contains, besides the real numbers, infinite numbers (larger in absolute value than any positive real number) and infinitesimal numbers (smaller in absolute value than any positive real number). In von Neumann-Bernays-Gödel set theory, the surreal numbers are the largest possible ordered field; all other ordered fields, such as the rationals, the reals, the rational functions, the Levi-Civita field, the superreal numbers, and the hyperreal numbers, can be obtained as subfields of the surreals.

Martin Gardner made a spectacular and conceptually accurate and appropriate comment: “It is an astonishing feat of legerdemain. An empty hat rests on a table made of a few axioms of standard set theory. Conway waves two simple rules in the air, then reaches into almost nothing and pulls out an infinitely rich tapestry of numbers that form a real and closed field. Every real number is surrounded by a host of new numbers that lie closer to it than any other ‘real’ value does. The system is truly *surreal*” [Gardner, *Mathematical Magic Show*, pp. 16--19]. And he continues ”I believe it is the only time a major mathematical discovery has been published first in a work of fiction.” Gardner was referring to Donald Knuth’s 1974 novelette *Surreal Numbers: How Two Ex-Students*

Turned onto Pure Mathematics and Found Total Happiness; not only was this discovery published in Knuth's novelette, but also the very name "surreal numbers" was proposed (and eventually accepted by Conway himself, instead of "numbers").Subsequently, Conway described the surreal numbers and used them in game analysis in his 1976 book *On Numbers and Games*. Surreal numbers redefined our general understanding of fundamental concepts such as *number* and *game*.

Knot Theory − the mathematical study of the properties of knots– also benefited from Conway's innovative ideas. His notation for knots is, in fact, an illuminating way of identifying knots in terms of their two-dimensional components, the tangles, which he studied early in his career. Conway completed the knot tables up to 10 crossings and introduced the Conway knot, a new knot with 11 crossings that cannot be obtained from a combination of simpler knots. Deciding whether or not two knot diagrams represent the same knot is a main problem in knot theory; it is solved by using knot polynomials, which are invariants of the knot. John Conway showed (1969) that a version of the *Alexander polynomial*, the Conway-Alexander polynomial, can be computed using *skein relations*. From 1984, with the discovery of the Jones polynomial, Conway's idea and results turned into a flourishing topic relating algebraic and geometric properties of knots. Jones-Conway polynomials generalize both Conway-Alexander polynomials and Jones polynomials

Combinatorial game theory, a theory of partisan games (where some moves are available to only one player), was introduced by John Conway with Elwyn Berlekamp and Richard Guy, with whom he also co-authored the book *Winning Ways for Your Mathematical Plays* (1982). Conway's book *On Numbers and Games* (1976) lays out the mathematical foundations of combinatorial game theory. For partisan games, the Sprague–Grundy theorem cannot be used, which makes them more difficult to analyze than impartial games. On the other hand, using combinatorial game theory to analyze partisan games makes it possible to grasp the full significance of *numbers as games* (*surreal numbers*), which does not happen for impartial games. Over a hundred partisan games are discussed in *Winning Ways for Your Mathematical Plays*, almost all of them invented by John Conway and his team.

Theoretical Physics. In 2006, John Conway and Simon Kochen published in *Foundations of Physics* a paper on their *free will theorem*. Forty years earlier, Bell's no-go theorem showed the gap between quantum mechanics and classical mechanics in terms of hidden variables: "No physical theory of local hidden variables can ever reproduce all of the predictions of quantum mechanics" (C.B. Parker, *McGraw-Hill Encyclopaedia of Physics*, 2nd edition, 1994). The Kochen–Specker theorem further restricted the kinds of hidden variable theories that would transfer the apparent randomness of quantum mechanics to a deterministic model with hidden states. The Conway-Kochen theorem provides a stunning version of the no-hidden-variables principle. It is based on three axioms, the first two of which are testable in quantum mechanics: one on the limitation of the speed for information propagation, one on the structure of the squared spin component of certain elementary particles, and the third that is a consequence of the quantum entanglement. In everyday language, the theorem states that, under the circumstances described by the axioms, if two experimenters are free to decide what measurements they make then the outcome of the measurements cannot be determined by things that happened before the experiment took place. In Conway's provocative wording: "If experimenters have free will, then so do elementary particles." (In a subsequent paper, Conway and Kochen weakened the first axiom, thus strengthening their result into *The Strong Free-Will Theorem.*) Such a startling result catalyzed a new facet of a debate, one that had begun with the Einstein–Bohr polemic, among physicists, philosophers and mathematicians, a debate which is most likely to illustrate a major "paradigm shift" in Kuhn's sense.

There are many other creations of John Conway that deserve a proper comment rather than this all too brief enumeration:

- in **Algebra**, particularly on quaternions (a number system that extends the *complex numbers*), icosians (a particular set of Hamiltonian quaternions invented with Neil Sloane) and octonions (Conway and Smith, On quaternions and octonions: their geometry, arithmetic, and symmetry, 2003);
- in **Geometry** (the 64 convex uniform polychora, the grand antiprism**)**;
- in **Algorithmics**, starting from his 1971 book *Regular Algebra and Finite Machines*. John Conway's best known algorithm may be the Doomsday Algorithm for very fast calculation of the day of the week for "any" date; he proposed a similar method for calculating the phase of the moon. The very simple and spectacular (if inefficient) 14-fraction prime-number generator should also be mentioned.

Notations. Great mathematicians of all times have contributed to the development of mathematical notation as a communication tool for expressing coherently and systematically the results of their original work and for facilitating further developments. The citation for his Steele Prize states, "He has a rare gift for naming mathematical objects and for inventing useful mathematical notations." Conway's contributions to notations can be related to his own words: "My job is to simplify things" – a splendid echo of Constantin Brancusi's words: "Simplicity? It is solved complexity".

John Conway invented notations for numbers (Conway chained arrow notation for exceedingly large numbers), also in geometric topology (the Conway notation for tabulating knots) and in geometry (the Conway polyhedron notation for describing polyhedra), the "orbifold notation for surface groups" (a simple way to enumerate crystallographic, spherical and wallpaper groups, algebraic structures that satisfy additional geometric properties). We would also include here Conway's famous *Look and Say Sequence* (which lends itself to data compression): this sequence is actually defined as a notation that directly combines the rhetorical stage and the symbolic stage of mathematical notation.

John Conway sees himself primarily as a **professor**. In his own words: "If it sits down, I teach it; if it stands up, I will continue to teach it; but if it runs away, I maybe not be able to catch up." Conway's most frequent and iconic image is one where he is surrounded by young students, during a challenging question-and-answer session or a game-playing session. His 13 PhD students continued to work with him and pursue very successful careers.

Recognition of Conway's work. John Conway's outstanding contributions received not only admiration, but, as an expression of this, some of the most prestigious prizes in Mathematics.

In 1971, Conway was awarded the Berwick Prize by the London Mathematical Society. The Berwick Prize is awarded in recognition of "an outstanding piece of mathematical research published by the Society" in the eight years before the year of the award. Conway's paper on sporadic finite simple groups was published only a year and a half before the deadline for the 1971 prize; the short interval between publication and award dates illustrates the instant enthusiastic reaction of the mathematical community.

Ten years later, Conway was elected a fellow of the Royal Society of London. The Royal Society's motto from Horace's *Epistles,Nullius in verba* ("take nobody's word for it"), can be seen as an indirect description of John Conway's original, challenging and provocative results.

In 1987, John Conway became the first recipient of the Pólya Prize of the London Mathematical Society. According to the LMS regulations, "The Pólya Prize is awarded in recognition of outstanding *creativity* in, *imaginative exposition* of, or *distinguished contribution* to, mathematics within the United Kingdom"; not only was John Conway the first recipient of the prize, but one cannot help guessing that the main features of his work and results may have inspired those who established these conditions for its award. Conway is also a Fellow of the American Academy of Arts and Sciences (the American Academy); the members of this most prestigious learned society are among "the finest minds and most influential leaders".

In 1994, John Conway was a main invited speaker at the International Congress of Mathematicians. Since 1897, the invited speakers at these Congresses have been those whose contributions to Mathematics were held in particularly high esteem. John Conway spoke about “Sphere Packings, Lattices, Codes, and Greed ” – in an exposition that is rigorous but still best described by Martin Gardner’s words above on the magic of Conway’s constructs: a theorem given without proof is turned into an axiom and eventually into a definition, while the spectacular final results show that integral lexicographic codes (lexicodes) are sphere packings and that winning strategies for games, in particular the Nimgame, can be described in terms of lexicodes or in terms of *laminated lattices*.

The Nemmers Prize was established in 1994 by Northwestern University, envisioning the creation of a reward that would be as prestigious as the Nobel Prize in Mathematics, Economics, Medical Sciences and Music. It is worth mentioning that six of the 11 laureates of the companion Nemmers Prize in Economics subsequently received the Nobel Prize in Economics (Nobel Prize laureates are not eligible for the Nemmers Prize). The prize in Mathematics is awarded to “those with careers of outstanding achievement in the field of Mathematics as demonstrated by major contributions to new knowledge or the development of significant new modes of analysis.” In 1998, John Conway received this prize in Mathematics “for his work in the study of finite groups, knot theory, number theory, game theory, coding theory, tiling, and the creation of new number systems”. A statement from the citation for this third Nemmers Prize in Mathematics underlines the impact of his achievements in the mathematical community and beyond:

"Conway may well have the distinction of having more books, articles and web pages devoted to his creations than any other living mathematician."

He was also awarded the Leroy P. Steele Prize for Mathematical Exposition by the American Mathematical Society, in 2000.

In 2001, John Conway received the Joseph Priestley Award from Dickinson College. This award, established in memory of Joseph Priestley, discoverer of oxygen, has been presented yearly since 1952 "to a distinguished scientist whose work has contributed to the welfare of humanity ..., recognizing outstanding achievement and contribution to our understanding of science and the world." Conway received the award "for distinguished contributions to the field of applied and computational mathematics."

Conclusion. John Conway's nomination as a Fellow of the Royal Society, where he was elected as a member in 1981, states that John Conway is "a versatile mathematician who combines a deep combinatorial insight with algebraic virtuosity, particularly in the construction and manipulation of 'off-beat' algebraic structures which illuminate a wide variety of problems in completely unexpected ways. He has made distinguished contributions to the theory of finite groups, to the theory of knots, to mathematical logic (both set theory and automata theory) and to the theory of games (as also to its practice)".

The juxtaposition of the words from the Steele Prize Citation and the title of the forthcoming biography by *Siobhan Roberts, Genius at Play,* seems to put John Conway's groundbreaking achievements in Mathematics into the right perspective: "His joy in Mathematics is clearly evident." *Indeed, simple facts such as the lack of any new universal* *automaton since the invention of Game of Life or the delay with which what is now considered an obvious and intuitive number system – surreal numbers – was discovered, point to the idea that playfulness may be instrumental in Mathematics.*

At the forefront of Mathematics, John Horton Conway has solved open problems, asked intriguing and inspiring mathematical questions, expanded existing results and defined new mathematical worlds in a brilliantly creative, sharply rigorous and usually spectacular way. His profound mathematical discoveries and inventions combine intellectual elegance and unexpectedness, the highest level of abstraction, and fruitful applications; they are at the same time illuminating and challenging. His results are accessible to top-level mathematicians and are instrumental in their work, while the applications of his results, which sometimes helped surpass limitations of current technologies, are visible or available to the widest audience. John Horton Conway embodies a rare kind of personality in science, one who is at the same time acclaimed by the leading scientific institutions and mathematicians, admired by amateur mathematicians, and widely known among laymen.

Alma Mater Iassiensis, *Alexandru Ioan Cuza* University of Iaşi, has the joy and honour to solemnly confer the title of Doctor **Honoris Causa** to ***John Horton Conway****, John von Neumann Professor, Emeritus, at Princeton University, a leading mathematician of our time, an outstanding professor and a prolific promoter of Mathematics and science at large.*

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Iaşi, June 24th 2014