COURSE PROGRAMME

1. Information about the programme

1.1 University	**University “Alexandru Ioan Cuza” of Iaşi**
1.2 Faculty	**Faculty of Mathematics**
1.3 Department	**Mathematics**
1.4 Domain	**Mathematics**
1.5 Cycle	**Master**
1.6 Programme / Qualification	**Applied Mathematics (MAE1)**

2. Information about the course

2.1 Course Name	VARIATIONAL CALCULUS AND OPTIMAL CONTROL							
2.2 Course taught by	**Prof.dr. Catalin-George LEFTER**							
2.3 Seminary / laboratory taught by	**Prof.dr. Catalin-George LEFTER**							
2.4 Year	**I**	**2.5** Semester	**1**	**2.6** Type of evaluation	**E**	**2.7** Course type	**OB**	

** OB – Obligatory / OP – Optionally / F – Facultative*

3. Total Hours (estimated per semester and activities)

3.1 Number of hours per week	**4**	3.2 course	**2**	3.3. seminary/laboratory	**2**
3.4 Total number of hours	**56**	3.5 course	**28**	3.6. seminary/laboratory	**28**

Distribution	hours
Individual study using textbooks, course notes, bibliography items, etc.	**75**
Supplimentary study (library, on-line platforms, etc.)	**20**
Individual study for seminary/laboratory, homeworks, projects, etc.	**20**
Tutoring	**-**
Examination	**4**
Other activities..................................	**-**

3.7 Total hours of individual activity	**115**
3.8 Total hours per semester	**175**
3.9 Credit points	**7**

4. Pre-requisites

4.1 Curriculum	**Differential equations. Differential geometry.**
4.2 Competencies	**Operates with notions, methods, fundamental techniques in Mathematical Analysis, Licence level**

5. Conditions (if necessary)

5.1 Course	Lecture hall
5.2 Seminary / Laboratory	Seminar hall

6. Specific competencies acquired

Professional competencies	C1 Manipulating notions, methods and mathematical models, specific techniques and technologies in scientific calculus and applications in economy and informatics(1 credit) C2 Data processing, analysis and interpretation using mathematical, statistical and informatics tools(1 credit) C3 Being able to develop, test and validate algorithms; implementation in high level programming languages (2 credits) C4 Being able to construct and apply mathematical models for analyzing and simulating some phenomena and processes(1 credit) C5 Being able to develop, analyze and test computer systems and specific programming languages; being able to use them for solving problems in applied mathematics (1 credit) C6 Being able to analyze and interpret some economic processes and phenomena
Transversal competencies	CT1 Having a responsible attitude towards scientific research and teaching, being able to fully develop the personal potential in the professional career, respecting the principles of a rigorous and efficient work in order to fulfill complex tasks, respecting the ethical norms and principles in the professional activity CT2 Being able to work efficiently in a team and to coordinate and efficiently lead a team or an inter-disciplinary group CT3 Being able to make a selection of information resources and to use them efficiently, in Romanian or other language of international circulation, in order to develop the professional activity and adapt it to the demands of a dynamical society

7. Course objectives

7.1. General objective	This course is an introduction to the fundamental ideas in Calculus of Variations and in Control Theory and the links between them.
7.2. Specific objectives	If successfull at the final examination, students will have an understanding of the following conceprs: ▪ Euler-Lagrange equations, Hamiltonian systems and Hamilotonian formalism in Calculus of Variations. ▪ Control of linear systems: controllability, optimal control with quadratic cost functional. ▪ Geometric formulation of optimal control problems. ▪ Controllability of differential systems and orbit theorem ▪ Maximum principle of Pontriaghin and dynamic programming principle.

8. Contents

8.1	**Course**	**Teaching methods**	**Remarks** (number oh hours, references)
1.	Introductory notions. Examples of problems in Calculus of Variations. First variation of a functional . Euler-Lagrange equations.	Exposition/Interactive presentation	2 hours
2.	Second variation of a functional. Sufficient conditions.	Exposition/Interactive presentation	2 hours
3.	Hamiltonian systems. Hamiltonian formalism.	Exposition/Interactive presentation	2 hours

4.	Total variation of a functional. Problems with end points on manifolds, transversality conditions. Non-smooth extremals, Weierstrass-Erdmann conditions.	Exposition/Interactive presentation	2 hours
5.	Hailton-Jacobi equation. Jacobi method for integrating hamiltonian systems.	Exposition/Interactive presentation	2 hours
6.	Control of differential systems – an introduction. Linear systems – controllability.	Exposition/Interactive presentation	2 hours
7.	Linear systems with convex cost functional. Optimality conditions. Pontriaghin maximum principle.	Exposition/Interactive presentation	2 hours
8.	Dynamic programming equation. Linear equatyions with quadratic cost functional. Riccati equations.	Exposition/Interactive presentation	2 hours
9.	Stabilization of linear differential equations. Algebraic Riccati equations.	Exposition/Interactive presentation	2 hours
10.	Representation of differential equations on differentiable manifolds.	Exposition/Interactive presentation	2 hours
11.	Controllability of differential systems. Orbit theorem and consequences.	Exposition/Interactive presentation	2 hours
12.	Analytic systems. Integrability, Frobenius theorem.	Exposition/Interactive presentation	2 hours
13.	Attainable sets and optimal control problems. Geometric form of Pontriaghin maximum principle.	Exposition/Interactive presentation	2 hours
14.	Optimal control problems with free final time. Maximum principle for free time problems	Exposition/Interactive presentation	2 hours

Bibliography

Main references:

- **V.Barbu, Mathematical methods in optimization of differential systems, Kluwer Academic Publishers, Dordrecht, 1994**
- **I.M. Gelfand, S.V. Fomin, Calculus of variations, 2000**
- **Cătălin Lefter, Calculul variatiilor si control optimal, Editura Alexandru Myller 200**
- **Cătălin Lefter, Calculus of Variations and Control Theory – available on the personal web page.**

Supplementary references:

- **L.Hocking, Optimal control, Oxford University Press, 1991**
- **J. Zabczyk, Mathematical control theory; an introduction, Birkhauser, Boston, 2008.**

8.2	**Seminary / Laboratory**	**Teaching methods**	**Remarks** (number oh hours, references)
1.	Introductory notions. Examples of problems in Calculus of Variations.	Interactive presentation/Exercises/Dialogue	2 hours

	First variation of a functional . Euler-Lagrange equations.		
2.	Second variation of a functional. Sufficient conditions.	Interactive presentation/Exercises/Dialogue	2 hours
3.	Hamiltonian systems. Hamiltonian formalism.	Interactive presentation/Exercises/Dialogue	2 hours
4.	Total variation of a functional. Problems with end points on manifolds, transversality conditions. Non-smooth extremals, Weierstrass-Erdmann conditions.	Interactive presentation/Exercises/Dialogue	2 hours
5.	Hailton-Jacobi equation. Jacobi method for integrating hamiltonian systems.	Interactive presentation/Exercises/Dialogue	2 hours
6.	Control of differential systems – an introduction. Linear systems – controllability.	Interactive presentation/Exercises/Dialogue	2 hours
7.	Linear systems with convex cost functional. Optimality conditions. Pontriaghin maximum principle.	Interactive presentation/Exercises/Dialogue	2 hours
8.	Dynamic programming equation. Linear equatyions with quadratic cost functional. Riccati equations.	Interactive presentation/Exercises/Dialogue	2 hours
9.	Stabilization of linear differential equations. Algebraic Riccati equations.	Interactive presentation/Exercises/Dialogue	2 hours
10.	Representation of differential equations on differentiable manifolds.	Interactive presentation/Exercises/Dialogue	2 hours
11.	Controllability of differential systems. Orbit theorem and consequences.	Interactive presentation/Exercises/Dialogue	2 hours
12.	Analytic systems. Integrability, Frobenius theorem.	Interactive presentation/Exercises/Dialogue	2 hours
13.	Attainable sets and optimal control problems. Geometric form of Pontriaghin maximum principle.	Interactive presentation/Exercises/Dialogue	2 hours
14.	Optimal control problems with free final time. Maximum principle for free time problems	Interactive presentation/Exercises/Dialogue	2 hours

References:
Same as above

9. Coordination of the contents with the expectations of the community representatives, professional associations and relevant employers in the corresponding domain

The material in this course is chosen in order to offer the students a global understanding of the fundamental ideas studied in the courses of Theoretical mechanics, Differential geometry, Differential equations. It offers an introduction to the modern field of Control theory which is directly connected to the practical applications of Mathematics.

10. Assessment and examination

Activity	**10.1 Criteria**	**10.2 Modes**	**10.3 Weight in the final grade (%)**
10.4 Course	Knowledge and correct use of notions and fundamental results which are presented in the course.	Written and oral exam	67%
10.5 Seminary/ Laboratory	Correct identification of methods for solving exercises and problems. Demonstration of calculation skills	Intermediate written examination (written test)	33%
10.6 Minimal requirements **Final mark:** N=(L+2E)/3, **Minimal standard** L>=5, E>=5, N>=5 **L= mark for intermediate test** **E=mark for the written & oral exam.**			

Date
9.10.2023

Course coordinator
Prof.dr. Catalin-George Lefter

Seminary coordinator
Prof.dr. Catalin-George Lefter

Aproval date in the department

Head of the departament
Prof. Dr. Ioan Bucătaru